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Neutrinos and the dark matter problem

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Theoretical and experimental arguments suggest that the mean mass density of our universe is close to the closure value and that most of the mass in the universe consists of weakly interacting non-baryonic particles. Among the plethora of candidates that have been proposed as the dark matter, the neutrino remains the only particle known to exist, even though the issue of a neutrino mass remains unresolved. It was shown several years ago that neutrinos alone cannot provide the dark matter because physical processes in the early universe would have wiped out primordial density fluctuations on the scale of galaxies and below. The idea that cosmic strings or textures may seed galaxy formation in a neutrino-dominated universe has not yet been demonstrated to be viable. On the other hand, a model in which the bulk of the dark matter is cold and neutrinos with a mass of *ca.* 10 eV provide a *ca.* 30% contribution can, in principle, overcome many of the objections against the standard cold dark matter cosmogony. Although subject to the usual 'fine-tuning' criticism, these mixed dark matter models represent the best cosmological argument in favour of a non-zero rest mass for the neutrino.

1. Introduction

At early times, the universe was remarkably uniform. Direct evidence for this was provided last year by the COBE satellite which mapped the structure of the microwave background – the relic radiation emitted when the Universe was only a few hundred thousand years old. The departures from homogeneity detected by COBE amount to an r.m.s. temperature fluctuation of only about 1 part in 10^5 on angular scales greater than 10° (Smoot *et al.* 1992). On smaller scales, a variety of experiments have set upper limits to the temperature fluctuations of comparable amplitude (Gaier *et al.* 1992; Cheng *et al.* 1993). Yet, the nearby universe is patently far from uniform; maps of the galaxy distribution clearly show a wide variety of structures, from aggregates of a few tens to elongated superclusters of several thousand galaxies. Understanding how the universe evolved from its remarkable early simplicity to its present, highly structure state is the central problem of physical cosmology.

The 1980s saw considerable progress in attempts to understand the growth of cosmic structure. This resulted from a timely combination of new theoretical ideas, computational methods and observational discoveries. Some of the most influential ideas came from the particle physics community: the notion of an inflationary universe and the suggestion that the dynamically dominant dark matter may consist of weakly interacting elementary particles. The inflationary paradigm (Guth 1981, Linde 1982; Albrecht & Steinhart 1982) is a beautiful idea which, furthermore, has observable consequences. According to it, the universe underwent a brief period of exponential expansion driven by the vacuum energy of a scalar field. This idea solves several cosmological puzzles: it explains why the universe is as old as it is, why it

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appears nearly homogeneous on large scales, and why it is so clumpy on small scales. The rapid expansion gives rise to a flat geometry and thus allows a long-lived universe. By bringing different regions within our current horizon into causal contact early on, inflation enables them to have similar physical properties. Quantum fluctuations in the field that drives the inflation are amplified to macroscopic scales and may subsequently grow by gravitational instability into the clustering pattern characteristic of the present day distribution of galaxies (Peebles 1980). The amplitudes of primordial density fluctuations produced by inflation are gaussian distributed and scale-invariant.

The temperature anisotropies detected by COBE appear to have just the properties expected in the inflationary model: a power spectrum corresponding to scale-invariant fluctuations ($|\delta_k|^2 \propto k^n$, with $n = 1.1 \pm 0.5$, where $k \equiv 2\pi/\lambda$ denotes spatial frequency) and a gaussian distribution. There are, however, alternative mechanisms to generate primordial seeds which are also consistent with the COBE data. An elegant idea posits the existence of topological defects in the early universe arising from field ordering phase transitions associated with the spontaneous breaking of global symmetries (Brandenberger 1991; Turok 1991; and references therein). Defects such as strings or textures give rise to local density perturbations which may subsequently grow, as in the inflationary case, through gravity.

Whether produced by quantum processes or by topological defects, primordial density perturbations develop at a rate which is determined by the amount and nature of the dark matter. The presence of dark matter was inferred 50 years ago by Zwicky, who realized that it was required to explain the existence of gravitationally bound clusters of galaxies. By the early 1980s physicists and astronomers began to take seriously the possibility that the neutrino could have just the right mass to close the universe and, at the same time, a number of other weakly interacting elementary particles, such as photinos or axions, were put forward as dark matter candidates. This suggestion shaped much of the thinking of cosmologists for the rest of the decade, although a baryonic nature for the dark matter is by no means discounted (see Carr 1990). Of all the elementary particles that have been proposed, the neutrino remains the only *known* particle. We shall see below, however, that neutrinos are unlikely to constitute the bulk of the dark matter, although some contribution from neutrinos is possible and, in some ways, desirable.

In this article, I review various arguments bearing on the role that neutrinos may play in shaping the cosmic large-scale structure. In §2 I discuss the main argument in favour of non-baryonic dark matter. The general theory for the gravitational origin of large-scale structure is summarized in §§3 and 4. These develop the argument against a universe dominated by massive neutrinos. Mechanisms that might make a neutrino model viable by seeding galaxies with cosmic string or clumps of cold dark matter are discussed in §5. A summary and conclusions are given in §6.

2. The mean density of the universe

One of the most dramatic successes of the hot Big Bang theory has been its ability to explain the relative abundances of the light elements. Thus current estimates of the 'primordial' abundances of H, ^{2}H, ^{3}He, ^{4}He, and ^{7}Li are all consistent with those expected a few minutes after the Big Bang in a Friedman–Robertson–Walker universe with present baryon density in the range:

$$0.01 \leqslant \Omega_b h^2 \leqslant 0.015. \tag{1}$$

where h is the present value of Hubble's constant in units of 100 km s^{-1} Mpc^{-1}, Ω_b is given in units of the critical density needed to close the universe and the range quoted represents a 95% confidence limit (Walker *et al.* 1991).

Over the last decade the cosmology community has increasingly accepted the plausibility of the inflation paradigm and of its associated prediction that the mean matter density of the universe, Ω, should be very close to the critical value for closure. The discrepancy between $\Omega = 1$ and $\Omega_\text{b} h^2 \approx 0.0125$ is the main motivation for the hypothesis that most of the mass in the universe is in some non-baryonic and invisible form.

For a long time direct dynamical evidence for such large amounts of dark matter was lacking. However, recent comparisons of large-scale motions of galaxies and clusters with the density field that induces them have consistently led to large estimates for Ω (Bertschinger *et al.* 1990; Rowan-Robinson *et al.* 1990; Kaiser *et al.* 1991; Strauss *et al.* 1992; Nusser & Dekel 1993). For example, analysis of the 'QDOT' redshift survey if IRAS galaxies (Kaiser *et al.* 1991; Frenk *et al.* 1994) gives:

$$\Omega^{-0.6} b'_I = 1.3 \pm 0.20, \tag{2}$$

where the biasing parameters, b'_I, is defined as the ratio of QDOT galaxy fluctuations to mass fluctuations in the same region, $b'_I = (\delta N/N)/(\delta\rho/\rho)$. Note that the dynamical analyses do not allow the determination of Ω independently of b. However, in most models of galaxy formation we expect galaxies to form preferentially in high density regions and so to be at least as, and probably more strongly clustered than the mass, corresponding to $b'_I \gtrsim 1$. Thus, the most straightforward interpretation of the QDOT result is that $\Omega = 1$ and $b'_I = 1.3 \pm 0.2$.

The result $\Omega = 1$ is of such fundamental importance that caution is well exercised. The samples used in the dynamical analyses are still small and may be subject to as yet ill-understood systematic effects. A further reason for caution was recently put forward by White *et al.* (1993*b*). These authors argue that the large mass fraction in baryons seen in rich galaxy clusters (about 17% for $h = 0.5$) compared with the low baryon density allowed by Big Bang nucleosynthesis (equation (1)) can only be explained if either Ω is low or the nucleosynthesis limit is incorrect. For most of this article, I will continue to assume that $\Omega = 1$, but we must bear in mind that this remains a controversial assumption.

3. The linear evolution of density fluctuations

There are many excellent reviews of this topic (White 1985; Efstathiou 1991), so I will not dwell on it here.

(*a*) *Initial conditions*

The primordial density and velocity fields are the initial conditions required to model the formation of large-scale structure. Density fluctuations can be of two types, adiabatic if the total energy density is perturbed, or isocurvature if only the energy density of some species of matter is perturbed. The fluctuations generated by inflation are adiabatic since they arise from fluctuations in the energy density of the quantum field which contains most of the energy of the universe. The structure of the density field is usually expressed in terms of its power spectrum, $|\delta_k|^2$, and a prescription for the phases of the fluctuations. The phases can be either random, in which case the mass in randomly placed spheres has a gaussian distribution with dispersion *ca.* $k^3|\delta_k|^2$, or correlated as in the case of strings or textures. Since quantum fluctuations in a free field have random phases, the perturbations that come out of inflation are gaussian. Once a density field has been specified, the associated velocities in linear theory are easily derived using Zel'dovich's (1970) formalism.

As the universe evolves, the primordial spectrum is distorted since different waves evolve in different ways. In the linear régime, i.e. while the amplitude of the fluctuations remains small, $|\delta_k|^2 \propto (1+z)^{-1}$, and every wave evolves independently of the others according to a transfer function, $T(k,t)$, which describes how each mode evolves with time. In this way, the primordial spectrum is propagated forwards in time:

$$\delta_k(t) = T(k,t)\,\delta_k^{\mathrm{p}}, \tag{3}$$

where δ_k^{p} is the primordial power spectrum, for example, the $n=1$ Harrison–Zel'dovich spectrum from inflation.

The form of the transfer function depends on the contents of the universe and their interactions which together determine the damping mechanisms that operate. Here, I briefly summarize the case in which the dark matter is made of weakly interacting elementary particles and the initial density fluctuations are as predicted in inflation: gaussian, adiabatic, and scale-invariant. There are two possibilities to consider.

(*b*) *Hot dark matter*

Hot dark matter consists of light particles which retain significant thermal motions for an extended period. The best example are neutrinos with a mass of *ca.* 30 eV. Their abundance is essentially fixed at the time when the neutrinos cease to be in thermal equilibrium with the radiation bath: $n_\nu = 4n_\gamma/11 \approx 100\ \mathrm{cm}^{-3}$. While the temperature of the universe remains above *ca.* $(30/K)$ eV, the neutrinos are relativistic. Any fluctuations that come within the horizon during this epoch are wiped out by free-streaming, a relativistic version of Landau damping in collisionless fluids. There is a critical wavelength,

$$\lambda_\nu = \frac{2\pi}{k_{\mathrm{c}}} = 41\left(\frac{m_\nu}{30\ \mathrm{eV}}\right)^{-1}\mathrm{Mpc} = \frac{13}{\Omega h^2}\mathrm{Mpc}, \tag{4}$$

shortwards of which fluctuations are damped by thermal motions (Bond *et al.* 1980; Bond & Szalay 1983). If the primordial spectrum had the inflationary shape, the post-recombination spectrum would be like that shown in figure 1.

(*c*) *Cold dark matter*

In this case, thermal motions are never important. This would occur if, as in the case of photinos, the dark matter particles are very heavy ($m_x \gtrsim 1$ GeV) or if, as in the case of axions, the particles are created with negligible momentum. After the temperature of the universe has fallen below m_x/K, and while the particles remain in equilibrium, their abundance drops relative to that of photons by the Boltzmann factor $\mathrm{e}^{m_x/KT}$. This process continues until the universe reaches the 'freeze out' temperature for the particle, at which the annihilation rate drops below the expansion rate. Since thermal motions are never important, free streaming is negligible. The main damping mechanism is the Mézáros effect whereby the growth of matter fluctuations is stiffled during the radiation era when the dominant photon-baryon fluid undergoes acoustic oscillations (Guyot & Zel'dovich 1970; Mézáros 1974). This effect produces a bend in the spectrum from the initial power-law index n to $n-4$, at a characteristic scale $\lambda_{\mathrm{c}} \approx 13(\Omega h^2)^{-1}$, corresponding to the horizon size

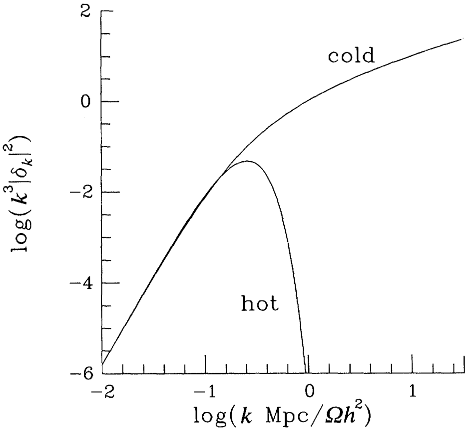


Figure 1. Power per decade as a function of spatial frequency for density fluctuations in universes dominated by weakly interacting elementary particles. These are linear power spectra evolved from the adiabatic constant curvature fluctuations predicted by inflation. Hot and cold dark matter differ in the magnitude of the random velocities at early times. The curves are taken from the calculations by Bond & Szalay (1983) and Bond & Efstathiou (1984).

at the epoch when the energy density in matter becomes equal to that in radiation. The cold dark matter spectrum is shown in figure 1.

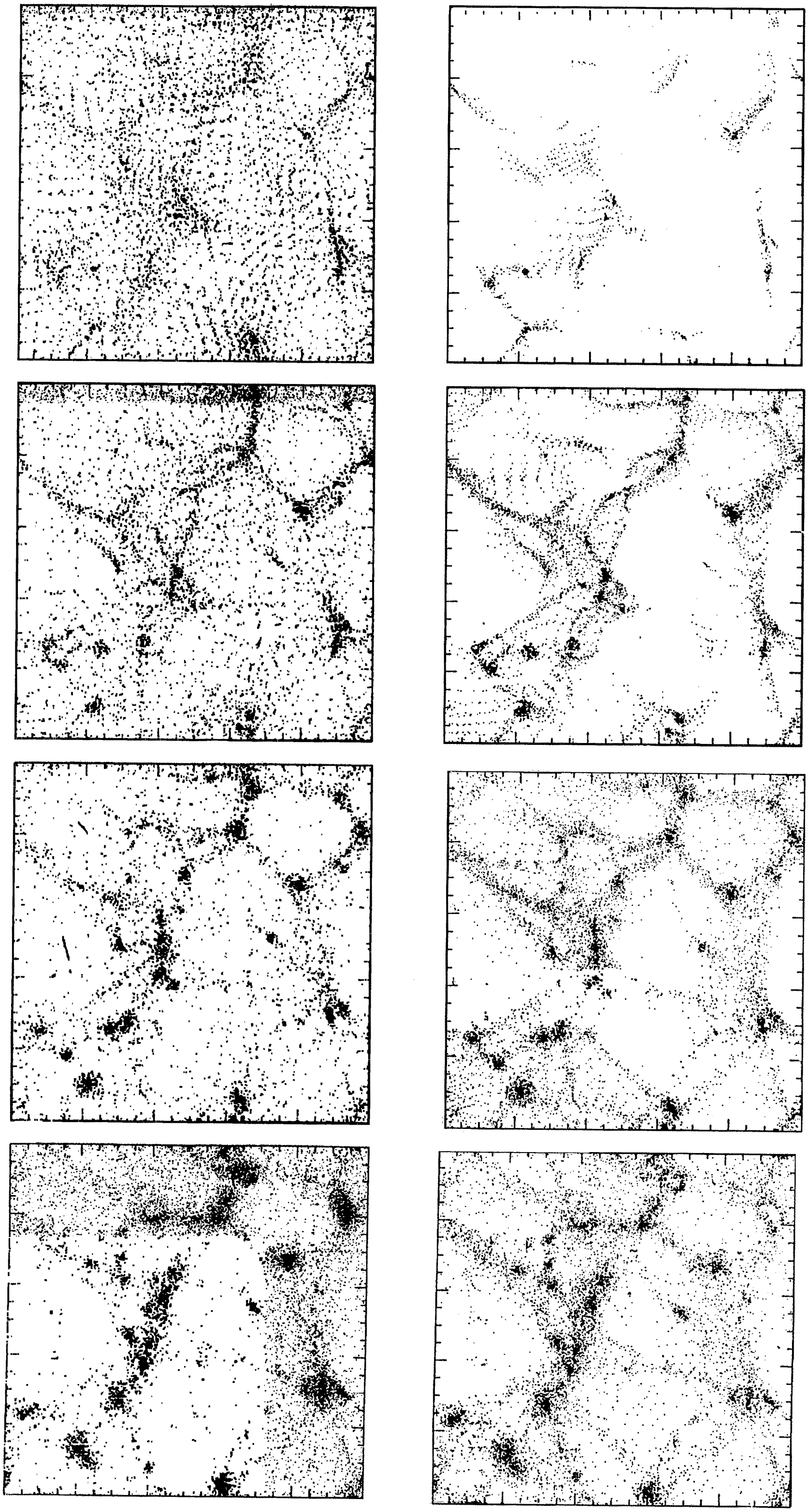
(*d*) *Normalization of the power spectrum*

For a given cosmological model, there is only one free parameter in figure 1: the fluctuation amplitude at some fiducial epoch. Until recently this had to be fixed empirically using, for example, the amplitude of the galaxy clustering pattern. The COBE results have changed this. Temperature fluctuations on large angular scales are produced by the Sachs–Wolfe effect (Sachs & Wolfe 1967), a general relativistic change in wavelength as photons traverse evolving regions of varying mass density. The amplitude of these fluctuations can be readily related to the amplitude of the mass fluctuations on the largest scales shown in figure 1 (see Bond & Efstathiou 1984; Holtzmann 1989; Wright *et al.* 1992; Efstathiou *et al.* 1992). Thus, in principle, the quadrupole anistropy measured by COBE may be used to normalise the power spectra of figure 1. There is a potential complication, however; in many inflationary models, tensor modes associated with long-wavelength gravitational waves produce a quadrupole anisotropy indistinguishable from that produced by density perturbations via the Sachs–Wolfe effect (Salopek 1992; Crittenden *et al.* 1993). If tensor modes are important, the inferred mass fluctuation amplitude would be overestimated.

Figure 1 summarizes the main impact of particle physics on studies of large-scale structure. Together with the assumption of random phases, it provides a set of definite predictions for the density field at the recombination era. These predictions still need to be propagated to the present in order to be confronted with observations, but this is mostly a matter of computation, not of principle.

4. The nonlinear evolution of density fluctuations

From figure 1 it is already clear that we expect cosmic structures to form in a different order depending on whether the dark matter is hot or cold. In the former case, all fluctuations with mass smaller than *ca.* $10^{15}/(\Omega h^2)^2 M_{\odot}$ have been erased by



the time of recombination, so there are no seeds from which galaxies can grow. Instead, superclusters are the first objects to decouple from the overall expansion and collapse, generally into flattened objects reminiscent of Zel'dovich's (1970) 'pancakes'. The baryons trapped in these sheets will shock and must subsequently cool and fragment in order for galaxies to form. By contrast, in a cold dark matter universe, there is power on all scales, with amplitude increasing with frequency. Subgalactic units are the first to collapse and they subsequently cluster hierarchically to produce large-scale structure. The nonlinear phases of the growth of structure are best studied using N-body simulations.

(*a*) *The neutrino dominated cosmogony*

When they were first discussed in the early 1980s, neutrino models seemed very attractive. A mass measurement near the value needed to close the universe had just been reported (Lyubimov *et al*. 1980); the excessive microwave background anisotropies predicted in purely baryonic models could be avoided (Bond *et al*. 1980); and, for good measure, a preferred scale of *ca*. $40h^{-1}$ Mpc, reminiscent of the largest superclusters known, was singled out by fundamental physics (equation (4)). It therefore came as a great disappointment that on closer examination the idea did not hold out. The arguments against neutrino dark matter have been reviewed by White (1985) and Frenk (1986, 1991), so I will limit this discussion to a brief summary.

The formation of large-scale structure in a neutrino universe is driven by the sharp cutoff in the fluctuation spectrum at a present-day scale of a few tens of magaparsecs (figure 1; Frenk *et al*. 1983; White *et al*. 1983, 1984). This can be seen in the left-hand column of figure 2, which shows the time evolution of the mass distribution in one of the N-body simulations of White *et al*. (1983). At the first epoch shown, the model has expanded by a factor of 4 from the initial conditions. Large scale filaments of length comparable to the critical wavelength are already apparent. As the model expands, the filaments become increasingly prominent giving rise to a characteristic 'beehive' pattern. Matter flows along these filaments and accumulates at their intersections so that later on the filaments begin to break up. Eventually, the mass distribution becomes completely dominated by a few large blobs which contain most of the mass. More recent and bigger simulations show much the same behaviour (Centrella *et al*. 1988; Zeng & White 1991; Cen & Ostriker 1992).

To compare the models with observations, we must decide where to place galaxies in the simulations. White *et al*. (1983) assumed that galaxies could only form in regions which have undergone local collapse. This is a very general requirement and represents a necessary (but not sufficient) condition for galaxy formation. The possible sites of galaxies are shown in the right-hand panels of figure 2. The galaxies follow the overall distribution of the mass, but the contrast of the structures which they delineate is greatly enhanced. The final consideration is which epoch in the simulations should be compared with the real world. This is equivalent to specifying the amplitude of the fluctuation spectrum at some fiducial time. In the pre-COBE

Figure 2. Projection onto two dimensions of the three-dimensional particle distribution in an N-body simulation of a neutrino dominated universe. The initial conditions (not shown) consist of 32768 particles perturbed from a uniform grid by waves with random phase and the power spectrum appropriate to a neutrino dominated universe. The size of the box is five times the critical wavelength of equation (4). The left-hand column shows the mass distribution and the right-hand column the 'galaxy' distribution defined as described in the text. From top to bottom, the different panels correspond to expansion factors 4, 6.1, 10, 20 respectively. Co-moving coordinates are used.

days, this was a free parameter and Davis, White and I argued that we could date the simulations by the epoch of galaxy formation. In the real world we observe quasars and radio-galaxies at redshifts $z > 3$, so collapsed structures must have already existed at that time. If we (conservatively) assume that the onset of galaxy formation in the simulations (defined as the time when 1% of the particles have passed through a collapsed region) occurred at $z = 2.5$, the present day should be identified with expansion factor 6.1 in figure 2. An artificial catalogue of 'galaxies' constructed from the simulations at this time, assuming similar geometry and selection effects as those in real surveys, is compared in figure 3 with a catalogue of real galaxies. It is immediately apparent that the model galaxies are much too strongly clustered.

If we use the COBE quadrupole anistropy to normalize the initial fluctuation spectrum in the simulations, the present day occurs only at expansion factor 4, very soon after the onset of 'galaxy formation'. Indeed, if this time were identified with the present, galaxy formation would have only began at a hopelessly recent redshift, $z = 0.4$. An artificial 'galaxy' catalogue based on this normalization is shown in the top panel of figure 3. Although still too strongly clustered, this distribution is a much better match to the real galaxy distribution than the model with $z_{gf} = 2.5$. But, of course, since we observe galaxies at redshifts much greater than 0.4, this model is completely ruled out. (If gravity waves contribute to the COBE quadrupole anistropy, the initial fluctuation amplitude would be even smaller than assumed here and the epoch of galaxy formation would be even more recent.)

To summarize, the problem with a neutrino universe is that galaxies cannot form until wavelengths of order a few tens of megaparasecs have collapsed. The small fluctuation amplitude measured by COBE implies that such wavelengths have only began to collapse recently, much too late to account for the observed presence of galaxies and quasars at even moderate redshifts. The obvious way to try and patch up a neutrino model is by introducing an external mechanism to seed galaxy formation at high redshift. Some such mechanisms have been proposed, but before discussing them, let us turn our attention to an alternative model which is considerably more successful.

(b) *The standard cold dark matter cosmogony*

Immediately after the demise of the neutrino model, the alternative proposition that the dark matter consists of cold, weakly interacting particles, such as supersymmetric particles or axions, began to be explored in detail (Peebles 1984; Blumenthal *et al.* 1984; Davis *et al.* 1985). In due course, this CDM model became the best studied and, in many ways, the most successful cosmogonic theory developed to date. This model has been reviewed extensively (Frenk 1991, 1992*a*, *b*; Davis *et al.* 1992) so I will restrict attention here to a few general remarks which lead into a possible connection to massive neutrinos.

The general properties of the mass distribution in the CDM model on scales ranging from those of galactic halos to those of clusters and superclusters were calculated in a series of N-body studies (Davis *et al.* 1985; Efstathiou *et al.* 1985; White *et al.* 1987*a*, *b*; Frenk *et al.* 1985, 1988, 1990; Gelb 1992; and other references in Frenk 1991). In the standard version of the model, galaxies are assumed to form only near the peaks of a suitably smoothed version of the linear density field – the 'high peak ansatz' (Davis *et al.* 1985; Bardeen *et al.* 1986). This model accounts for a wide variety of observed properties of galaxies, clusters, and their distribution on small

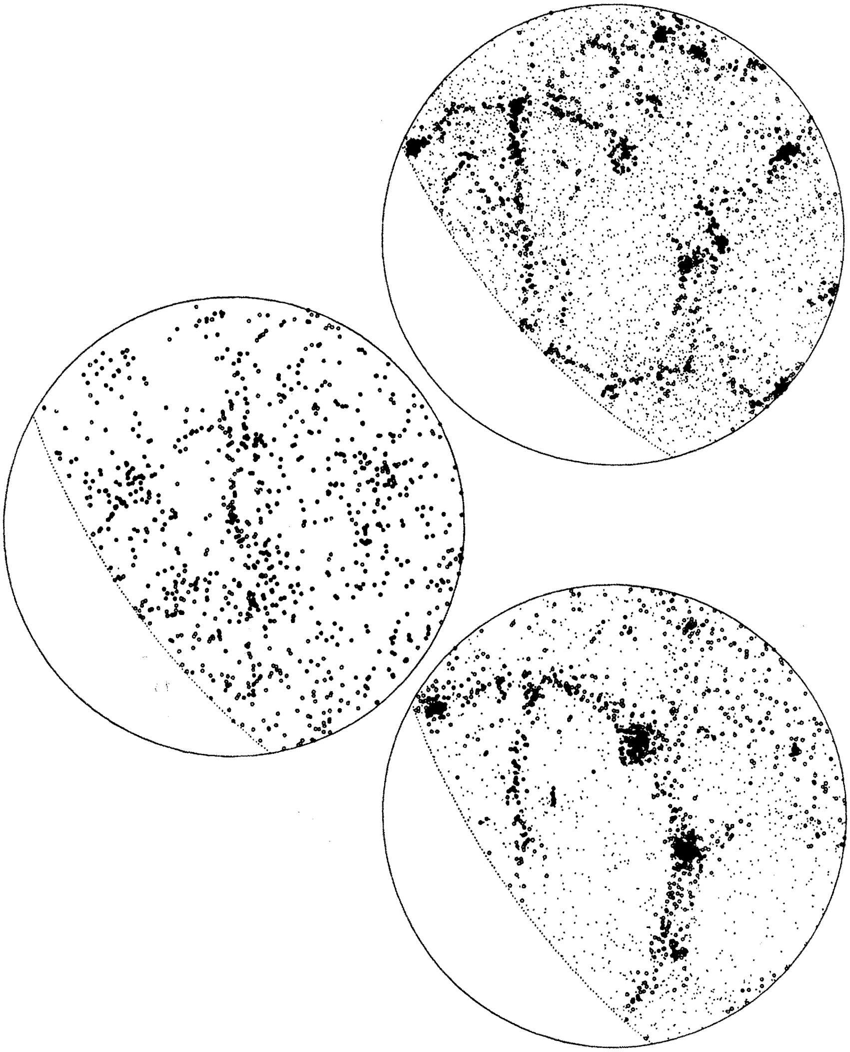


Figure 3. Equal area projections of the galaxy distributions on the northern sky (left) and in artificial catalogues made from N-body simulations (right). The two diagrams on the right correspond to neutrino dominated universes in which galaxy formation began at a redshift 0.4 (top) and 2.5 (bottom). In both cases $\Omega = 1$, but $h = 0.8$ for the model on the top, and $h = 0.5$ for the model on the bottom. The first of these matches the normalization implied by the COBE quadrupole anisotropy. The circles represent the 'galaxies' while the dots represent the neutrino distribution. The left-hand diagram is the CfA northern survey. The outer circle represents galactic latitude +40, and the empty regions lie at declinations below 0°.

and intermediate scales. However, on the largest scales for which quantitative studies have been made, the standard model predicts significantly weaker clustering than observed in the galaxy distribution (Maddox *et al.* 1990; Efstathiou *et al.* 1990; Saunders *et al.* 1991; Vogeley *et al.* 1992; Loveday *et al.* 1992; Fisher *et al.* 1993; Moore *et al.* 1992). Several modifications of the theory have been proposed to account for this 'excess power'. Some appeal to our ignorance of how galaxies formed and suggest a more complicated formation mechanism than the clearly oversimplified high peak model. An example is the 'cooperative model' of Bower *et al.* (1993) which provides an excellent match to the large-scale data while preserving the successes of the CDM model on small scales. Other proposals invoke a modification of the power spectrum and thus the introduction of additional physics into the model. Some of these involve a revival of the neutrino dark matter hypothesis patched up to circumvent the problem discussed above. I now turn to a brief review of some of these ideas.

5. Massive neutrinos revisited

The key ingredient lacking in a simple neutrino dominated model is a mechanism to trigger galaxy formation before the collapse of $10^{15} M_\odot$ superclusters. Let us consider two of the most promising possibilities that have been suggested.

(a) *Neutrinos and cosmic strings*

Cosmic strings (or other topological defects such as textures) can seed the formation of structure by creating density inhomogeneities in their vicinity (Brandenberger (1991) and references therein). In the current version of the string model, high surface density wakes left behind by long pieces of string as they move about at very high speed act as seeds for matter accretion and are ideal candidates for triggering early galaxy formation in a neutrino cosmogony.

Albrecht & Stebbins (1992) have calculated the fluctuation spectrum in a flat, string-seeded universe in which the dark matter consists of massive neutrinos. The string wakes produce power on small scales by collecting neutrinos after the free-streaming epoch is over. Nevertheless, the spectrum still declines on scales smaller than the critical wavelength of equation (4), so that this model remains essentially a ‘top-down’ cosmogony. The crucial feature, however, is that the seeds produce a non-gaussian distribution of density fluctuations. Galaxy sized clumps on the tail of the distribution may still collapse early enough, before the critical wavelength has become nonlinear. Unfortunately, this is a difficult process to model and no detailed calculations yet exist to demonstrate that the correct abundance of long-lived galaxies can be produced.

As in the Gaussian case, the fluctuation amplitude in string-seeded models may be fixed by comparison with the COBE quadrupole anisotropy. The normalization can be expressed in terms of the one free parameter in the theory: the mass per unit length on the string, $G\mu$. Stebbins (personal communication) finds $G\mu = 10^{-6}$ and a biasing parameter $b \approx 1-2$. The resulting spectrum, however, has even less power on scales $\gtrsim 20\ \mathrm{h}^{-1}$ Mpc than the standard cold dark matter model. Thus the string-seeded neutrino cosmogony may face the same difficulties as the standard model in accounting for the large measured amplitude of galaxy clustering. Neutrino models in which the seeds are promoted by the presence of textures rather than strings do not fare much better (Cen *et al.* 1993).

(b) *Neutrinos and cold dark matter*

As we saw earlier, the CDM cosmogony is a success story on galactic scales where the neutrino cosmogony fails, but appears to be wanting on large scales where neutrino fluctuations may be important. The pragmatist is thus strongly tempted to propose a model with a mixture of the two. Indeed, shortly after the COBE discovery, there was a spate of papers discussing just these ‘mixed dark matter’ (MDM) models (Davis *et al.* 1992; Taylor & Rowan-Robinson 1992; Klypin *et al.* 1993; Pogosyan & Starobinsky 1993). About 60–80 % of the dark matter in these models is cold and the

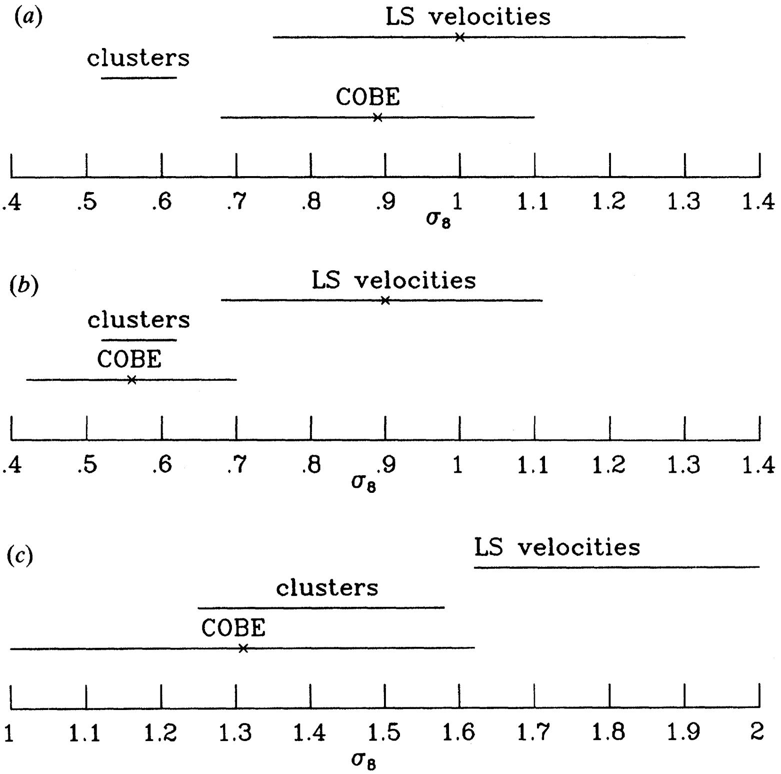


Figure 4. Constraints on the amplitude of mass fluctuations in three models of large-scale structure. The inferred values of the amplitude at 8 h^{-1} Mpc (σ_8) are shown, as derived from the COBE quadrupole anisotropy, large-scale galaxy flows, and the abundance of rich galaxy clusters. The crosses indicate the best fit value of σ_8 and the lines its $\pm 1\sigma$ uncertainty, except in the 'clusters' case where the errors are likely to be dominated by systematics. The COBE value includes a 20% downward correction to allow for small departures from the Harrison–Zel'dovich spectrum and a minimal contribution from gravity waves as predicted in standard models of inflation. The three models tested are (*a*) the standard cold dark matter model, (*b*) a mixed hot (neutrinos) and cold dark matter model, and (*c*) a flat, low density cold dark matter model case. The parameters assumed in each case are: (*a*) $\Omega = 1$, $h = 0.5$; (*b*) $\Omega_{\mathrm{CDM}} = 0.8$, $\Omega_{\nu} = 0.2$, $h = 0.5$; (*c*) $\Omega_{\mathrm{CDM}} = 0.2$, $\Omega_{\Lambda} = 0.8$, $h = 1$.

remainder hot. The net effect of the mixture is to slightly retard the growth of fluctuations on scales on which the neutrino component is too hot to cluster. This occurs for wavelengths smaller than the neutrino Jeans length, *ca.* $0.8(1+z)^{\frac{1}{2}}(m_{\nu}/10\ \mathrm{eV})\ h^{-1}$ Mpc, corresponding, roughly, to galaxies today. Thus, for a fixed fluctuation amplitude on 8 h^{-1} Mpc, the flatter spectral shape on small scales translates into additional power on large scales.

The mixed dark matter model has many attractive features. It can simultaneously match the COBE quadrupole amplitude and the 'excess' clustering of galaxies on scales *ca.* 30 h^{-1} Mpc. On smaller scales, it produces a cooler velocity field and thus more realistic pairwise galaxy peculiar velocities than the CDM model. Furthermore, the right mixture of dark matter components is achieved if there is one species of neutrino with mass *ca.* 10 eV, which, as discussed elsewhere in this volume, fits in nicely with the proposed solution to the solar neutrino problem and the seesaw model of neutrino masses. These undeniable advantages have to be weighed against the niggling worry that a new component has been introduced into the simplest model, with an attendant free parameter (the ratio of hot to cold dark matter) carefully tuned to match the experimental data. But perhaps we must accept the fact that the universe is not as simple as it might have been!

Fortunately, there are additional tests to which the MDM and indeed other models can be subjected. Bearing in mind that the galaxy distribution may be masking non-gravitational effects such as those involved in the cooperative galaxy formation model mentioned in §4*b*, the most robust tests are those which measure the amplitude of *mass* fluctuations directly. There are two well-studied mass diagnostics, in addition to microwave background temperature fluctuations: the large scale streaming motions of galaxies and the abundance of galaxy clusters. Peculiar velocities (Lynden-Bell *et al.* 1988) measure the contrast relative to the background of the mass lumps that induce them, while the abundance of massive objects like clusters is a direct reflection of the inhomogeneity of the mass distribution. The way in which these two diagnostics are applied is discussed by Frenk *et al.* (1990), Efstathiou *et al.* (1992), and White *et al.* (1993*a*).

Figure 4 summarizes the constraints from the mass diagnostics on three currently popular models of large scale structure: the standard CDM model (SCDM), a mixed dark matter model (MDM) and a flat CDM model with a low value of Ω and a non-zero cosmological constant. For each model, the bars show the range in σ_8 (the rms linear fluctuation amplitude on 8 h^{-1} Mpc) allowed by each diagnostic. For a model to be consistent with the measured COBE quadrupole anistropy, large scale peculiar velocities and abundance of massive clusters, all three error bars should overlap. The MDM model seems to give a marginally better fit than the other two models. However, given the relatively large observational uncertainties, it would clearly be rash to exclude any of the models shown on the basis of this analysis. It seems clear, however, that improved determinations of the relevant observational quantities will lead to a stronger conclusion.

7. Conclusions

New ideas in fundamental physics coupled with technical innovations in theoretical and observational tools during the past decade have given unprecedented impetus to cosmological studies. The long awaited detection of primordial fluctuations in the microwave background radiation represents one of the most important advances in observational cosmology for many years. In a broad brush sense, it confirms the general picture of structure formation from the gravitational growth of small primordial perturbations. At a more detailed level, the COBE detection allows an estimate of the amplitude of primordial density fluctuations.

One of the most influential ideas to have emerged from the particle physics connection is the inflationary model of the early universe which predicts a flat geometry and provides a mechanism to generate primordial density fluctuations which have a gaussian distribution and a power spectrum consistent with the COBE detection. The theoretically appealing idea of a universe with critical density has recently received some empirical support through the comparison of large-scale motions of galaxies with the density field that induces them. This idea, together with the low abundance of baryons allowed by Big Bang nucleosynthesis leads to the view that the dark matter in our universe consists of weakly interacting, non-baryonic elementary particles.

Neutrinos with a mass of *ca.* 30 eV are *a priori* the most attractive particle candidates for the dark matter. Simulations of the formation of structure, however, show that this model is not viable – galaxies do not form early enough to accord with observation. Cosmic strings provide a mechanism to trigger early galaxy formation in a neutrino dominated model, but detailed calculations are difficult to carry out and are, as yet, inconclusive. Furthermore, with the normalization implied by COBE, a universe with neutrinos and strings may fail to reproduce the large-scale inhomogeneity observed in the galaxy distribution.

A cosmogonic model invoking cold dark matter – weakly interacting particles which, unlike neutrinos, have low thermal velocities at early times – is successful in reproducing many of the properties of galaxies and clusters and their distribution on intermediate spatial scales. Recent surveys of galaxies on larger scales, however, suggest that observed superclusters have a higher contrast, by about a factor 2, than predicted in this standard model. Several modifications have been proposed to promote a more efficient formation of large structures. One idea is that superclusters may reflect a large-scale modulation of galaxy properties (produced, for example, by the effects of quasar radiation fields on the pregalactic environment) rather than a true inhomogeneity in the mass distribution. Other suggestions involve changing the primordial mass distribution. One possibility is to invoke a *ca.* 30% contribution to the dark matter in the form of neutrinos with a mass of *ca.* 10 eV. Although these 'mixed dark matter' models are subject to the usual 'fine-tuning' criticism, they do fix up some longstanding problems of the standard model. At this time, this idea provides the strongest cosmological argument in favour of a non-zero neutrino mass.

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